Math 270 Day 5 Part 1

Section 2.3: First-Order Linear Differential Equations

Section 2.3: First-Order Linear Differential Equations

What we'll go over in this section

- What is a first-order linear differential equation
- How to solve first-order linear differential equations
- Why this method for solving first order linear differential equations works

Section 2.3: Separable Differential Equations

What is a first-order linear differential equation?

Definition: A first-order linear differential equation is a differential equation that can be written in the form

$$a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$$

Note: The standard form for a first-order linear differential equation is

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Section 2.3: Separable Differential Equations

Solving a first-order linear differential equation

To solve a first-order linear differential equation $a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$ (it's like completing the square)

1) Move terms around (if necessary) so that the $\frac{dy}{dx}$ and y terms are on the left and everything else is on the right

2) Get rid of the coefficient of the $\frac{dy}{dx}$ term by dividing both sides by whatever is in front of it (divide by $a_1(x)$)

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (*)$$

3) Calculate the integrating factor $\mu(x) = e^{\int P(x)}$

4) Multiply both sides of equation (*) by the integrating factor

Section 2.3: Separable Differential Equations

Solving a first-order linear differential equation

To solve a first-order linear differential equation $a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$ (it's like completing the square)

5) Now, the left side becomes $(\mu(x)y)'$

6) Integrate both sides

7) Solve for y

8) Check your solution

Section 2.3: Separable Differential Equations

Solving a first-order linear differential equation

Ex 1: Solve the DE $x^2 \frac{dy}{dx} - xy - 2x^4 = 0 \ , \ x > 0$

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Solving a first-order linear differential equation

Ex 2: Solve the IVP $\frac{dy}{dx} - 2y = xe^{3x} - e^{2x}\cos x \ , \ y(0) = 3$

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Solving a first-order linear differential equation

Ex 3: Find the integrating factor for the DE $\left(1-x^2\right)\frac{dy}{dx}-x^2y=(1+x)\sqrt{1-x^2}$

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Solving a first-order linear differential equation

Ex 4: Find the integrating factor for the IVP $(\cos x)\frac{dy}{dx} + y\sin x = 2x\cos^2 x \quad , \quad y\left(\frac{\pi}{4}\right) = -\frac{15\sqrt{2}\pi^2}{32}$

Section 2.3: Separable Differential Equations

Why this method for solving first order linear differential equations works